Economic Order Quantity

Reference: Hopp, Wallace J., and Mark L. Spearman. Factory physics. Waveland Press, 2011.

Econonic order quantity (EOQ) expresses the fundamental tradeoff between setup costs and holding costs. Small lot sizes have large setup costs per unit, but large lot sizes incur holding costs.

The variables are:

```
D = Variable("D", "count/year", "Demand rate")
c = Variable("c", "USD/count", "per unit production cost")
A = Variable("A", "USD", "setup cost")
h = Variable("h", "USD/count/year", "holding cost")
Q = Variable("Q", "count", "lot size")
Y = Variable("Y", "USD/year", "cost per year")
```

The cost per year is simply the sum of holding, setup, and production costs,

$$Y(Q) = \frac{hQ}{2} + \frac{AD}{Q} + cD$$

Or in gpkit,

```
eoq = [Y >= h*Q/2 + A*D/Q + c*D]
```

Now create a model that minimizes cost and substitute in the values given in Factory Physics.

```
from gpkit import Model, units
m = Model(Y, eoq)
m.substitutions.update({D: 1000,  # /units("year"),
                        c: 250,   # *units("USD"),
                        A: 500,   # *units("USD"),
                        h: 35})   # *units("USD/year")})
sol = m.solve()
# would like to assert here that
# sol.subinto((2*A*D/h)**0.5 =~= sol.subinto(Q)
```

Free Variables	Value	Units	Description
Q	169	count	lot size
Y	2.559e+05	$\frac{\text{USD}}{\text{year}}$	cost per year

Constants	Value	Units	Description
A	500	USD	setup cost
D	1000	$\frac{\text{count}}{\text{year}}$	Demand rate
c	250	$\frac{\text{USD}}{\text{count}}$	per unit production cost

h	35	$\frac{\text{USD}}{(\text{count}\cdot\text{year})}$	holding cost

Sensitivities	Value	Units	Description
D	+0.99	$\frac{\text{count}}{\text{year}}$	Demand rate
c	+0.98	$\frac{\text{USD}}{\text{count}}$	per unit production cost
A	+0.012	USD	setup cost
h	+0.012	$\frac{\text{USD}}{(\text{count}\cdot\text{year})}$	holding cost

Now sweep Q to see the tradoff curve from Factory Physics

```
import numpy as np
m.substitutions.update({
    Q: ("sweep", np.logspace(np.log10(10), np.log10(500), 25))})
sol = m.solve()
```

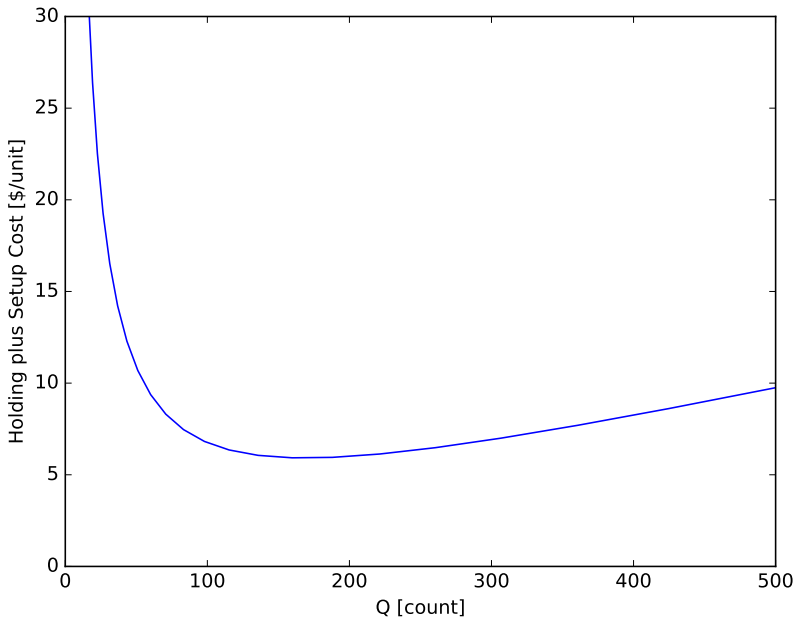

Figure 1: Cost vs Order Quantity